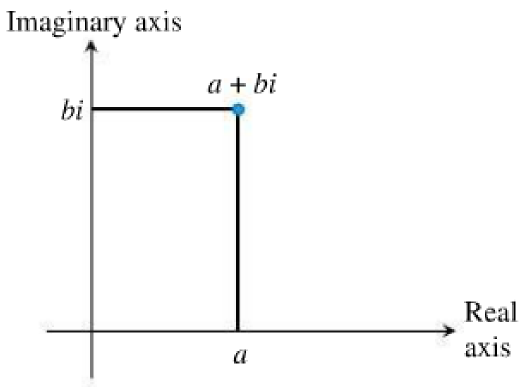
Section 6.6 NOTES DeMoivre's Theorem and nth roots

The Complex Plane:



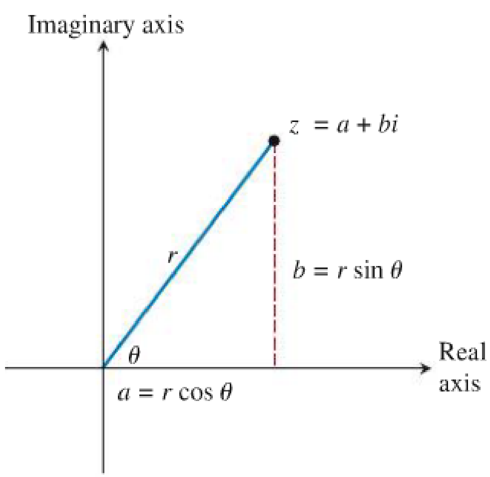


FIGURE 6.59 If r is the distance of $z = a + bi$ from the origin and θ is the directional angle shown, then $z = r(\cos \theta + i \sin \theta)$, which is the polar form of z.

DEFINITION Polar Form of a Complex Number

The **polar form** of the complex number $z = a + bi$ is

$$z = r(\cos \theta + i \sin \theta)$$

where $a = r \cos \theta$, $b = r \sin \theta$, $r = \sqrt{a^2 + b^2}$, and $\tan \theta = b/a$. The number r is the *absolute value* or *modulus* of z, and θ is an **argument** of z.

In Exercises 13–18, write the complex number in standard form $a + bi$.

13. $3(\cos 30° - i \sin 30°)$

14. $8(\cos 210° + i \sin 210°)$

15. $5[\cos(-60°) + i \sin(-60°)]$

16. $5\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

17. $\sqrt{2}\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)$

18. $\sqrt{7}\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$

De Moivre's Theorem

Let $z = r(\cos \theta + i \sin \theta)$ and let n be a positive integer. Then

$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta).$$

EXAMPLE 5 Using De Moivre's Theorem

Find $(1 + i\sqrt{3})^3$ using De Moivre's Theorem.

SOLUTION

Solve Algebraically See Figure 6.63. The argument of $z = 1 + i\sqrt{3}$ is $\theta = \pi/3$, and its modulus is $|1 + i\sqrt{3}| = \sqrt{1 + 3} = 2$. Therefore,

$$z = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

$$\begin{aligned} z^3 &= 2^3\left[\cos\left(3 \cdot \frac{\pi}{3}\right) + i \sin\left(3 \cdot \frac{\pi}{3}\right)\right] \\ &= 8(\cos \pi + i \sin \pi) \\ &= 8(-1 + 0i) = -8 \end{aligned}$$

Support Numerically Figure 6.64a sets the graphing calculator we use in complex number mode. Figure 6.64b supports the result obtained algebraically.

In Exercises 31–38, use De Moivre's Theorem to find the indicated power of the complex number. Write your answer in standard form $a + bi$.

31. $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^3$

32. $\left[3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right]^5$

33. $\left[2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^3$

34. $\left[6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^4$

35. $(1 + i)^5$

36. $(3 + 4i)^{20}$

37. $(1 - \sqrt{3}i)^3$

38. $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^3$